FRACTIONAL EQUATIONS

A ***fractional equation*** is an equation with one or more fractions in it. Since any fraction is really a division problem, it seems reasonable that we could solve such an equation by multiplying each side of the equation by something. What should that something be? The ***LCD*** (Least Common Denominator) of all the denominators in the equation works perfectly.

❑ *SOLVING FRACTIONAL EQUATIONS*

EXAMPLE 1: **Solve for z:** $\dfrac{z+6}{4} - \dfrac{z+4}{6} = \dfrac{2}{3}$

Solution:

First step: Determine the LCD; it's **12**. Reason: All three denominators, the 4, the 6, and the 3, divide into 12, AND 12 is the smallest number that does this.

Second step: Multiply each side of the equation by the LCD. [If this is done correctly, all denominators will be eradicated.]

Third step: Distribute, cross-cancel, and solve the equation.

Multiply each side of the equation by 12, the LCD:

$$12\left(\frac{z+6}{4} - \frac{z+4}{6}\right) = 12\left(\frac{2}{3}\right)$$

Distribute (and write 12 as $\frac{12}{1}$ if you'd like):

$$\frac{12}{1}\left(\frac{z+6}{4}\right) - \frac{12}{1}\left(\frac{z+4}{6}\right) = \frac{12}{1}\left(\frac{2}{3}\right)$$

Cross-cancel, thus removing all fractions from the equation:

$${}^{3}\frac{\cancel{12}}{1}\left(\frac{z+6}{\cancel{4}_{1}}\right) - {}^{2}\frac{\cancel{12}}{1}\left(\frac{z+4}{\cancel{6}_{1}}\right) = {}^{4}\frac{\cancel{12}}{1}\left(\frac{2}{\cancel{3}_{1}}\right)$$

$$3(z + 6) - 2(z + 4) = 4(2)$$

Now solve for z in the usual way:

$$3z + 18 - 2z - 8 = 8$$

$$z + 10 = 8$$

$$\boxed{z = -2}$$

CHECK: Let $z = -2$ in the original equation. Do NOT do any algebra; just do simple arithmetic on each side:

$\frac{z+6}{4} - \frac{z+4}{6}$	$\frac{2}{3}$
$\frac{\mathbf{-2}+6}{4} - \frac{\mathbf{-2}+4}{6}$	
$\frac{4}{4} - \frac{2}{6}$	
$1 - \frac{1}{3}$	
$\frac{2}{3}$	✓

Homework

1. Solve and check each equation:

a. $\frac{m}{5}+\frac{m}{4} = \frac{6}{5}$

b. $\frac{b}{2}-\frac{b}{6} = \frac{1}{3}$

c. $\frac{n}{3}-\frac{n}{2} = 1$

d. $\frac{r}{2}-\frac{r}{3} = \frac{1}{3}$

2. Solve and check each equation:

a. $\frac{m-4}{6} + \frac{m+6}{5} = 2$

b. $\frac{b-4}{5} - \frac{b+6}{4} = 3$

c. $\frac{c+3}{3} + \frac{c+5}{2} = \frac{1}{3}$

d. $\frac{k+4}{2} - \frac{k-4}{3} = 1$

EXAMPLE 2: **Solve for *x*:** $\boldsymbol{\frac{2}{x}+\frac{y}{z} = w}$

Solution: The LCD is xz (remember, the w can be written $\frac{w}{1}$). Therefore, we shall multiply each side of the equation by $\boldsymbol{xz}$:

$\frac{2}{x}+\frac{y}{z} = w$ (the original equation)

$\Rightarrow$ $xz\left(\frac{2}{x}+\frac{y}{z}\right) = xz(w)$ (multiply each side by xz, the LCD)

$\Rightarrow$ $\cancel{x}z\left(\frac{2}{\cancel{x}}\right)+x\cancel{z}\left(\frac{y}{\cancel{z}}\right) = xz(w)$ (distribute and cross-cancel)

$\Rightarrow$ $2z + xy = wxz$ (simplify)

$\Rightarrow$ $xy - wxz = -2z$ (x's to the left; the rest to the right)

$\Rightarrow$ $x(y - wz) = -2z$ (factor out the x, the GCF)

$\Rightarrow$ $\boxed{x = \frac{-2z}{y-wz}}$ (divide each side by $y - wz$)

Homework

3. Solve for x: $\dfrac{a}{b} - \dfrac{x}{a} = c$

4. Solve for y: $\dfrac{y}{t} + \dfrac{c}{d} = \dfrac{u}{w}$

5. Solve for n: $\dfrac{c}{h} + \dfrac{w}{n} = \dfrac{b}{x}$

6. Solve for z: $\dfrac{a}{z} - \dfrac{x}{n} = m$

7. Solve for c: $\dfrac{b}{m} + \dfrac{z}{c} = \dfrac{n}{w}$

8. Solve for n: $\dfrac{u}{n} + \dfrac{m}{t} = d$

EXAMPLE 3: **Solve for d:** $\mathbf{7 - \dfrac{6}{d-2} = \dfrac{1}{d-2}}$

Solution: The LCD is $d - 2$, so this is what we'll multiply each side of the equation by:

$$\left[\frac{d-2}{1}\right]\left(7 - \frac{6}{d-2}\right) = \left[\frac{d-2}{1}\right]\left(\frac{1}{d-2}\right)$$

$$\Rightarrow (d-2)(7) - \cancel{(d-2)}\left(\frac{6}{\cancel{d-2}}\right) = \left(\frac{\cancel{d-2}}{1}\right)\left(\frac{1}{\cancel{d-2}}\right) \quad \text{(distribute)}$$

$$\Rightarrow \quad 7(d-2) - 6 = 1 \qquad \text{(cross-cancel)}$$

$$\Rightarrow \quad 7d - 14 - 6 = 1 \qquad \text{(distribute)}$$

$$\Rightarrow \quad 7d - 20 = 1 \qquad \text{(simplify)}$$

$$\Rightarrow \quad 7d = 21 \qquad \text{(add 20)}$$

$$\Rightarrow \quad \boxed{d = 3} \qquad \text{(divide by 7)}$$

CHECK: Put 3 in for d in the original equation:

$7-\dfrac{6}{d-2}$	$\dfrac{1}{d-2}$
$7-\dfrac{6}{\mathbf{3}-2}$	$\dfrac{1}{\mathbf{3}-2}$
$7-\dfrac{6}{1}$	$\dfrac{1}{1}$
$7-6$	1
1	✓

The sides balance, so $d = 3$ is the solution.

EXAMPLE 4: **Solve for u:** $\mathbf{1-\dfrac{16}{u}+\dfrac{28}{u^2} = 0}$

Solution: The LCD is u^2. Multiply each side of the equation by u^2:

$$u^2\left(1-\frac{16}{u}+\frac{28}{u^2}\right) = u^2(0)$$

$$\Rightarrow \quad u^2(1)-u^2\left(\frac{16}{u}\right)+u^2\left(\frac{28}{u^2}\right) = u^2(0) \quad \text{(distribute)}$$

$$\Rightarrow \quad u^2 - 16u + 28 = 0 \quad \text{(cross-cancel)}$$

$$\Rightarrow \quad (u-14)(u-2) = 0 \quad \text{(factor)}$$

$$\Rightarrow \quad u-14 = 0 \text{ or } u-2 = 0 \quad \text{(set each factor to 0)}$$

$$\Rightarrow \quad \boxed{u = 14 \text{ or } u = 2}$$

You should check these two potential solutions to ensure that both work in the original equation.

<u>EXAMPLE 5:</u> **Solve for *m*:** $\dfrac{\boldsymbol{m}}{\boldsymbol{m}+\mathbf{9}} + \mathbf{8} = \dfrac{\mathbf{-9}}{\boldsymbol{m}+\mathbf{9}}$

<u>Solution:</u> We begin as usual by multiplying each side of the equation by the LCD; in this problem, it's $m + 9$:

$$[m+9]\left(\frac{m}{m+9}+8\right) = [m+9]\left(\frac{-9}{m+9}\right)$$

$$\Rightarrow \quad [m+9]\frac{m}{m+9}+[m+9]8 = [m+9]\frac{-9}{m+9} \qquad \text{(distribute)}$$

$$\Rightarrow \quad \cancel{[m+9]}\frac{m}{\cancel{m+9}}+[m+9]8 = \cancel{[m+9]}\frac{-9}{\cancel{m+9}}$$

$$\Rightarrow \quad m + 8m + 72 = -9 \qquad \text{(cross-cancel)}$$

$$\Rightarrow \quad 9m + 72 = -9 \qquad \text{(combine like terms)}$$

$$\Rightarrow \quad 9m = -81 \qquad \text{(subtract 72)}$$

$$\Rightarrow \quad \underline{m = -9} \qquad \text{(divide by 9)}$$

Now for the **check**; put -9 in for m in the <u>original</u> equation:

$\dfrac{m}{m+9}+8$	$\dfrac{-9}{m+9}$
$\dfrac{\mathbf{-9}}{\mathbf{-9}+9}+8$	$\dfrac{-9}{\mathbf{-9}+9}$
$\dfrac{-9}{0}+8$	$\dfrac{-9}{0}$

Hold it right there!! We have zero on the bottom of both fractions. Continuing with this calculation is meaningless, since we have two undefined fractions. We've reached the end of our check. Does this mean we made a mistake in solving the equation? Absolutely not! We did everything according to the rules, so the equation must be inherently flawed. Thus, our only candidate for a solution, -9, has failed to be a solution. We have no choice but to admit that no number will make the original equation true. Our conclusion:

$\boxed{\text{No solution}}$ (Some teachers would write this as the null set: ∅)

EXAMPLE 6: **Solve for x:** $\dfrac{5}{x+1} - \dfrac{3}{x+3} = 4$

Solution: Multiply each side of the equation by the LCD: $(x + 1)(x + 3)$. Then distribute, cross-cancel, and solve the resulting quadratic equation by factoring.

Multiply each side of the equation by the LCD, $(x + 1)(x + 3)$:

$$(x+1)(x+3)\left(\frac{5}{x+1} - \frac{3}{x+3}\right) = (x+1)(x+3)(4)$$

Distribute the LCD, and then cross-cancel:

$$\cancel{(x+1)}(x+3)\left(\frac{5}{\cancel{x+1}}\right) - (x+1)\cancel{(x+3)}\left(\frac{3}{\cancel{x+3}}\right) = (x+1)(x+3)(4)$$

$$5(x + 3) - 3(x + 1) = 4(x + 1)(x + 3)$$

Distribute on the left and double distribute on the right:

$$5x + 15 - 3x - 3 = 4(x^2 + 4x + 3)$$

Combine like terms on the left, and distribute on the right:

$$2x + 12 = 4x^2 + 16x + 12$$

Since we have a quadratic equation, bring all terms to one side:

$$0 = 4x^2 + 14x$$

Turn the equation around and factor:

$$2x(2x + 7) = 0$$

Set each factor to 0:

$$2x = 0 \text{ or } 2x + 7 = 0$$

Solve each linear equation:

$$\boxed{x = 0 \text{ or } x = -\frac{7}{2}}$$

CHECK: I'm tired; you check both solutions.

EXAMPLE 7: **Solve for y:** $\dfrac{y-2}{y+1} = \dfrac{y+7}{y-8}$

Solution: Multiply each side of the equation by the LCD:

$$(y+1)(y-8)\left(\frac{y-2}{y+1}\right) = (y+1)(y-8)\left(\frac{y+7}{y-8}\right)$$

Cross-cancel the common factors on each side of the equation:

$$\cancel{(y+1)}(y-8)\left(\frac{y-2}{\cancel{y+1}}\right) = (y+1)\cancel{(y-8)}\left(\frac{y+7}{\cancel{y-8}}\right)$$

And simplify:

$$(y-8)(y-2) = (y+1)(y+7)$$

Notice that this equation can be found directly from the original equation by "cross-multiplying."

Double distribute on each side of the equation:

$$y^2 - 10y + 16 = y^2 + 8y + 7$$

Subtract y^2 from each side of the equation:

$$-10y + 16 = 8y + 7$$

Finally, solve for y in the standard way:

$$-18y + 16 = 7 \Rightarrow -18y = -9 \Rightarrow y = \frac{-9}{-18} = \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}}$$

CHECK:

$$\begin{array}{c|c} \dfrac{y-2}{y+1} & \dfrac{y+7}{y-8} \\[2ex] \dfrac{\mathbf{\frac{1}{2}}-2}{\mathbf{\frac{1}{2}}+1} & \dfrac{\mathbf{\frac{1}{2}}+7}{\mathbf{\frac{1}{2}}-8} \\[2ex] \dfrac{-\frac{3}{2}}{\frac{3}{2}} & \dfrac{\frac{15}{2}}{-\frac{15}{2}} \\[2ex] -1 & -1 \end{array} \checkmark$$

Homework

9. Solve and check each equation:

a. $\dfrac{4x}{x-8}+1 = \dfrac{9}{x-8}$

b. $\dfrac{1}{2n}+\dfrac{2}{n} = \dfrac{5}{6}$

c. $1-\dfrac{12}{a}-\dfrac{64}{a^2} = 0$

d. $g+\dfrac{24}{g} = -10$

e. $\dfrac{4}{t+1}-\dfrac{6}{t+3} = 2$

f. $\dfrac{9}{b-1}-\dfrac{9}{b+3} = 3$

g. $\dfrac{x+2}{x+4} = \dfrac{x+5}{x}$

h. $\dfrac{q-5}{-2} = \dfrac{-6}{q-1}$

i. $\dfrac{9}{x}-\dfrac{70}{x^2} = -1$

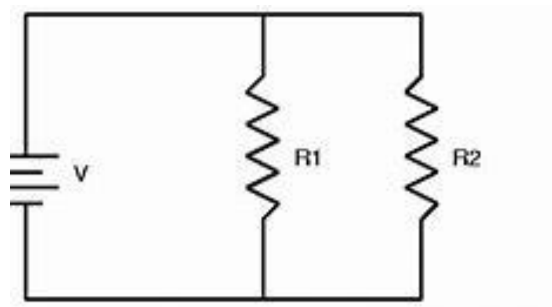
j. $\dfrac{-8}{g+1} = \dfrac{8g}{g+1}-9$

k. $\dfrac{1}{4a}+\dfrac{2}{5a} = \dfrac{8}{5}$

l. $\dfrac{2}{m+8}+\dfrac{1}{m+6} = 1$

❑ ELECTRONICS

The electrical circuit shows a voltage source along with two resistors wired in **parallel**. The TOTAL **resistance**, R_T, in the circuit is related to the resistors R_1 and R_2 by the formula



$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}}$$

EXAMPLE 8: **A parallel circuit contains two resistors, one with a resistance of $8\,\Omega$ and the other with a resistance of $5\,\Omega$. Find R_T, the total resistance in the circuit.**

> *Resistance* can be measured in a unit called the **ohm**, (named after Georg Ohm) and abbreviated using the Greek capital letter omega, Ω.

Solution: Using the formula we can write

$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{5}$$

The LCD is $40R_T$, the product of the three denominators. Multiplying both sides of the equation by the LCD gives

$$40R_T\left(\frac{1}{R_T}\right) = 40R_T\left(\frac{1}{8}\right) + 40R_T\left(\frac{1}{5}\right)$$

$$\Rightarrow \quad 40 = 5R_T + 8R_T$$

$$\Rightarrow \quad 40 = 13R_T$$

$$\Rightarrow \quad R_T = \frac{40}{13}$$

$$\Rightarrow \quad \boxed{R_T \approx 3.08\,\Omega}$$

Alternate Solution:

$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{R_T} = \frac{13}{40} \qquad \text{(add the fractions)}$$

$$\Rightarrow \quad R_T = \frac{40}{13} \qquad \text{(take the reciprocal of each side of the equation, giving the same answer)}$$

EXAMPLE 9: **Solve the total resistance formula for R_1.**

Solution: We start, of course, with the given formula:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

The LCD is the product of all three denominators, which is $R_T R_1 R_2$, so we multiply each side of the formula by the LCD, which clears out the fractions:

$$\left[R_T R_1 R_2\right]\left(\frac{1}{R_T}\right) = \left[R_T R_1 R_2\right]\left(\frac{1}{R_1}\right) + \left[R_T R_1 R_2\right]\left(\frac{1}{R_2}\right)$$

Now simplify by cross-canceling:

$$R_1 R_2 = R_T R_2 + R_T R_1$$

$$\Rightarrow \quad R_1 R_2 - R_T R_1 = R_T R_2 \qquad \text{(bring all } R_1\text{'s to the left side)}$$

$$\Rightarrow \quad R_1(R_2 - R_T) = R_T R_2 \qquad \text{(factor out our unkown } R_1\text{)}$$

$$\Rightarrow \quad \boxed{R_1 = \frac{R_2 R_T}{R_2 - R_T}} \qquad \text{(divide each side by } R_2 - R_T\text{)}$$

Homework

10. A parallel circuit contains two resistors, one with a resistance of $5\,\Omega$ and the other with a resistance of $0.2\,\Omega$. Find R_T, the total resistance in the circuit.

11. The total resistance in a parallel circuit containing two resistors is $\mathbf{8\,\Omega}$. If one of the resistors has a resistance of $\mathbf{10\,\Omega}$, find the resistance of the other resistor.

12. Solve the total resistance formula for R_T.

13. Solve the total resistance formula for R_2.

Review Problems

14. Solve for x: $\dfrac{x-2}{5} - \dfrac{2x-1}{6} = \dfrac{4}{15}$

15. Solve for x: $\dfrac{a}{y} - \dfrac{z}{x} = \dfrac{a}{b}$

16. Solve for a: $1 - \dfrac{25}{a} + \dfrac{150}{a^2} = 0$

17. Solve for x: $\dfrac{x}{x-3} + \dfrac{3}{2} = \dfrac{3}{x-3}$

18. Solve for x: $\dfrac{2}{x-3} + \dfrac{3}{x+1} = \dfrac{3}{2}$

Solutions

1. a. $m = \frac{8}{3}$ b. $b = 1$ c. $n = -6$ d. $r = 2$

2. a. $m = 4$ b. $b = -106$ c. $c = -\frac{19}{5}$ d. $k = -14$

3. $x = \frac{abc - a^2}{-b} = \frac{a^2 - abc}{b}$

4. $y = \frac{dtu - ctw}{dw}$

5. $n = \frac{-hxw}{cx - bh} = \frac{hxw}{bh - cx}$

6. $z = \frac{an}{mn + x}$

7. $c = \frac{-mwz}{bw - mn} = \frac{mwz}{mn - bw}$

8. $n = \frac{tu}{dt - m}$

9. a. $x = \frac{17}{5}$ b. $n = 3$ c. $a = 16, -4$

d. $g = -4, -6$ e. $t = 0, -5$ f. $b = 3, -5$

g. $x = -\frac{20}{7}$ h. $q = -1, 7$ i. $x = 5, -14$

j. No solution k. $a = \frac{13}{32}$ l. $m = -4, -7$

10. $R_T \approx 0.192\ \Omega$

11. $40\,\Omega$

12. $R_T = \frac{R_1 R_2}{R_1 + R_2}$

13. $R_2 = \frac{R_1 R_T}{R_1 - R_T}$

14. $x = -\frac{15}{4}$

15. $x = \frac{byz}{ab - ay}$

16. $a = 10, 15$

17. No solution

18. $x = 5, \frac{1}{3}$

“We cannot hold a torch

to light another's path

without brightening our own.”

– Ben Sweetland